Journal of Engineering Mathematics, Vol. 8, No. 4, October 1974
Noordhoff International Publishing – Leyden
Printed in The Netherlands

Comments

ON I. H. HERRON'S "COMMENTS ON A PAPER BY K. K. TAM: A NOTE ON THE FLOW IN A TRAILING VORTEX"

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(Received April 16, 1974)

I was amused to read Herron's summary in which he claimed that I suggested Batchelor's work was in error. Throughout his comments, he was extremely generous in attributing to me intentions and statements which are certainly not mine.

I shall first mention some such statements, then discuss the two points he raised. In what follows, VII-1 refers to Volume 7, Number 1 of this Journal.

(1) Herron: "Tam suggested that early work by Batchelor on the decay of a laminar trailing vortex was in error."—I made no such suggestion.

(2) Herron: "Batchelor's asymptotic solution was criticized by Tam, who attempted to refute its validity on the grounds of non-uniqueness." ... Tam (line 13–14, p. 3, VII-1): "Clearly then, within the framework of the linearized system, it is not possible to single out an appropriate solution without further information or constraints."

(3) Herron: "In equation (7a), $-K_1 g_1(\zeta)$ represents the velocity deficit on the axis. Tam stated it as a boundary condition but it is actually unknown." ......Tam (line 16, p. 4, VII-1): "The constant K_1 is a parameter of the problem, and $g_1(\zeta)$ is to be determined."

The above list is not meant to be exhaustive.

Now, let me consider the points raised by Herron: "Firstly, the method Tam used to attack the "arbitrariness" of Batchelor's solution may also be applied to his own. Secondly, the mass flux deficit given by his solution is not finite."

Herron's discussion on the first point is contained mainly in the three paragraphs following his equation (10). He used his equations (11) and (14) to show that his $\psi^{(1)}(\zeta, \eta)$ can assume different forms. Now, it must be quite obvious that when I constructed the asymptotic expansions for ψ and Ω, (line 17–39, p. 4, VII-1) no claim whatsoever was made about the uniqueness of the solution. Indeed, it was clearly explained that the choice of a particular form for $g_1(\zeta)$ was dictated by the desire to keep the Ω_0 term in (24). It is thus extremely difficult to understand why Herron would interpret this expansion procedure as my "method to attack the "arbitrariness" of Batchelor's solution". Possibly, Herron's confusion was a result of his equations (6a), (6b), which he attributed to me. He observed, incorrectly, that "Tam made a perturbation expansion about a free stream with a decaying vortex: (6a), (6b)", and hence he thought the perturbation, which he denoted by $\psi^{(1)}(\zeta, \eta)$, could take on the forms of his equations (11) and (14). In fact. my equations (21) are most definitely not his (6a), (6b). I used my equations (21) to introduce asymptotic expansions for ψ and Ω, for $\zeta \to \infty$. Herron's (6a), (6b) have no such meaning. His first point is therefore a result of his misinterpretation of my equations (21).

Herron's second point, which may seem to have some substance, is contained in the second last paragraph of his comments. This point concerns the fact that the mass flux $\psi_1(\eta)$ obtained by me is logarithmically infinite as $\eta \to \infty$, and thus Herron believes that $\psi_1(\eta)$ is not physically meaningful. Before proceeding further, I would like to refer Herron to Batchelor's paper, especially the discussion on p. 657. Now, in considering a single axisymmetric vortex, and using the condition that $\Omega \to 1$ as $\eta \to \infty$, we end up with $v_\theta = O(1/r)$ for large r, and so the kinetic energy $\int_0^\infty v_\theta^2 r\,dr$ is logarithmically infinite. This, however, does not affect the physical situation as trailing vortices occur as a pair, with opposite sense of rotation. A physically meaningful result can still be obtained by subtracting out the logarithmically divergent quantity in the outer field. Physically, this means that at large distances from the core, the presence of a pair of vortices ensures that Ω will decay, and does not tend to a constant as is supposed in the analysis. What happens to Ω ties in with what happens to ψ_1. If my equation (24) is integrated once, it yields

$$\eta^2(\psi_1''' + \psi_1'') = -\frac{T}{2}\,\Omega_0^2 .$$

Clearly, as long as $\Omega_0 \to 1$ as $\eta \to \infty$, we have $\psi'_1 = O(1/\eta)$ for large η, giving rise to a logarithmic term for ψ_1. Since physically Ω_0 must decay as $\eta \to \infty$, it is clear that for large η, the above equation behaves like

$$\eta^2(\psi_1''' + \psi_1'') = 0 ;$$

and so ψ_1 can remain finite. Therefore, a physically meaningful result can be obtained from ψ_1 by subtracting out the logarithmically divergent quantity, in the same way as Batchelor did to obtain the "drag associated with the core of a (single) trailing vortex". Herron's second point is thus of no consequence.

The last paragraph of Herron's comments is related to his first point and needs no further discussion.

In closing, I cannot help but claim that Herron's comments constitute no refutation whatsoever of my results. I also thank the Editor of this Journal for according me the courtesy of printing this reply together with Herron's comments.

Discussion closed (*The Editor*).